

Dynamics of Dark Matter in Baryon-Radiation Plasma: Perspectives using Meschersky equation

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With an aim to argue for the truly collisionless nature of cold dark matter between epochs of equality and recombination, we assume a model, wherein strongly coupled baryon-radiation plasma ejects out of small regions of concentrated cold dark matter without losing its equilibrium. We use the Meschersky equation to describe the dynamics of cold dark matter in the presence of varying mass of strongly coupled baryon-radiation plasma. Based on this model, we discuss the growth of perturbations in cold dark matter both in the Jeans theory and in the expanding universe using Newton's theory. We see the effect of the perturbations in the cold dark matter potential on the cosmic microwave background anisotropy that originated at redshifts between equality and recombination i.e. $1100 < z < z_{eq}$. Also we obtain an expression for the Sachs-Wolfe effect, i.e. the CMB temperature anisotropy at decoupling in terms of the perturbations in cold dark matter potential. We obtain similar solutions both in the static and in the expanding universe, for epochs of recombination. From this, we infer about the time scale when the dark energy starts to dominate.

Meschersky equation, Cold Dark matter, Collisionless, Baryon-Radiation plasma, Cosmic Microwave background, Sachs-Wolfe effect

I. INTRODUCTION

Flat cosmological models with a mixture of ordinary baryonic matter, cold dark matter, and cosmological constant(or quintessence) and a nearly scale-invariant, adiabatic spectrum of density fluctuations are consistent with standard inflationary cosmology. They provide an excellent fit to current observations on large scales($\gg 1$ Mpc). Currently, the constitution of the universe is 4% baryons, 23% dark matter and 73% dark energy [1–6].

In the standard hot Big Bang model, the universe is initially hot and the energy density is dominated by radiation. The transition to matter domination occurs at $z \approx 10^4$. In the epochs after equality and before recombination, the universe remains hot enough. Thus the gas is ionized, and the electron-photon scattering effectively couples the matter and radiation [7]. At $z \approx 1200$, the temperature drops below ≈ 3300 K. The protons and electrons now recombine to form neutral hydrogen and neutral helium. This event is usually known as recombination [8–10]. The photons then decouple and travel freely. These photons which keep on travelling till present times are observed as the cosmic microwave background(CMB). The cold dark matter theory including cosmic inflation is the basis of standard modern cosmology. This is favoured by the CMB data and the large scale structure data [11, 12]. The CDM model is based on the assumption that the mass of the universe now is dominated by dark matter, which is non-baryonic [13, 14]. Also it acts like a gas of massive, weakly interacting(collisionless)particles [15]. They have negligibly small primeval velocity dispersion. Also they are electromagnetically neutral [16, 17].

The word Cold here means that the ratio $\frac{T}{M} < \phi$, the gravitation potential, where T and M represent the temperature and mass of the dark matter particle. There is remarkably good agreement between standard CDM models and the observed power spectrum of Lyman α observers [18]. This rules out the warm dark matter candidates. The CDM model predicts the power spectrum of the angular distribution of the temperature of the 3K cosmic microwave background radiation and the flat Friedmann model. A low density CDM model with a density parameter of around 0.3 to 0.4 with cosmological constant actually matches all available data fairly well [19, 20]. The stable CDM paradigms predict all structure formation [21, 22].

The existence of clusters(≤ 50 mpc) and groups of galaxies suggests that the galaxy formation is due to the gravitational instability of a spatially homogenous and isotropic expanding universe. The Perturbations of such a model have been first investigated by [23, 24]. Then within the framework of general relativity by [25, 26]. Also it was studied in a newtonian model[32]. The results obtained from relativistic theory were similar to the standard Jeans theory. In the early universe, radiation fixes the expansion rate. This is due to low density and self-gravity of dark matter [28, 29]. After equality, the main contribution to the gravitational potential is due to the cold dark matter.

In the metric of the perturbed FRW universe, the main contribution to the gravitational potential comes from an

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imperfect fluid, i. e. the dark matter. We still consider that the difference $\psi - \phi$ is suppressed compared to ψ , atleast by the ratio of the photon mean free path to the perturbation scale. Here ψ and ϕ are the newtonian potential and the perturbations to the spatial curvature in a conformal Newtonian gauge respectively(see eq. 48 sec II). We neglect the contribution of the baryon density in realistic models, where baryon contributes only a small fraction of the total matter density. At $\eta > \eta_{eq}$, the gravitational potential is mainly due to the perturbations in the cold dark matter. The potential of cold dark matter is time-independent both for long wavelength and short wavelength perturbations. This is because it is highly non-relativistic. There is a strong coupling between the baryon and the radiation in the plasma. So, we treat it as a single perfect fluid for low baryon densities. We neglect the non-diagonal components in the energy- momentum tensor of dark matter. This is because for very small photon mean free paths ψ is same as ϕ . Therefore, we treat it as a single perfect fluid for many epochs after equality upto the recombination. It is only after recombination, that the baryon density starts increasing. This is after the primordial nucleosynthesis of hydrogen and helium is complete. The radiation after decoupling from matter at very late recombination epochs evolves separately. The CMB anisotropy of epochs between equality and recombination gives important information about the perturbations in the cold dark matter i. e. the modes that enter the horizon before recombination.

In our model, the strongly coupled baryon-radiation plasma ejects out of a concentrated region of cold dark matter. Based on this model, we study the dynamics of the cold dark matter after equality and upto many epochs near recombination. Note that this sort of flow assumes that the baryon -radiation plasma ejects out without losing its equilibrium in the presence of cold dark matter. This is possible only if we assume that the dark matter is truly collisionless. This assumption can also accomodate a self-interacting dark matter at very small scales, which can transfer energy and momentum to the outer core [30]. The strongly-coupled baryon -radiation plasma is only under the influence of the potential of cold dark matter at these epochs. We assume that the number of photons per Cold dark matter is initially spatially uniform on supercurvature scales(wavelengths greater than H^{-1}). During this time, the matter and radiation densities vary in space. In other words we consider adiabatic perturbations. As the universe expands, the inhomogeneity scale becomes smaller than the curvature scale. Thus the components move with respect to one another and the entropy of photon per cold dark matter particle varies spatially. In contrast the entropy per baryon remains spatially uniform on all scales. This is until the baryons decouple from radiation.

We use Meshchersky equation to study the dynamics of the strongly coupled baryon- radiation plasma after the cold dark matter starts to dominate. This occurs after epochs of equality. In this model, we imagine a flow of the baryon - radiation plasma across regions of highly concentrated non -relativistic cold dark matter. The baryon -radiation plasma can be assumed to have an ejection velocity with respect to the cold dark matter after equality. We do not disturb the equilibrium of the ejecting baryon-radiation plasma, even in the presence of cold dark matter. This we do to argue in favour of truly collisionless nature of cold dark matter. We ensure this by a specific assumption(see eq. 27 sec. II). This is the time, when the radiation densities rapidly start to decrease. This is due to its scaling proportional to a^{-4} , which is a^{-3} for the cold dark matter density. On the basis of this model, we study the adiabatic perturbations between epochs of equality and recombination. This we do, both in the Jean's theory and in an expanding universe in Newtonian theory. We then find out the effect of perturbations in cold dark matter potential on the anisotropy in temperature of radiation at these epochs. This we do for modes greater than the curvature scales. These are the modes which enter horizon before recombination.

The paper proceeds as follows. In section 2, we discuss the Dynamics of Cold dark matter in Baryon-radiation plasma using the Meshchersky equation in the Jeans theory. Then we discuss the adiabatic perturbations in the scope of this model. We also deduce an equation which represents how the anisotropy in the temperature of radiation is affected by the perturbations in the cold dark matter in the epochs between equality and recombination. We derive an expression for the Sachs-Wolfe effect. In Section 3, in the scope of our model, we discuss the dynamics of the above mentioned scenario in the expanding universe in the Newtonian theory. We show how the anisotropy in the temperature of radiation is affected by the perturbations in Cold dark matter potential, in an expanding universe. Also we evaluate an expression for the Sachs-Wolfe effect in the expanding universe. We then write the equation for the evolution of cold dark matter perturbations with time.

II. GRAVITATIONAL INSTABILITY: JEANS THEORY REPRESENTATION

In Jeans theory we consider a static, non-expanding universe. Also we assume a homogeneous, isotropic background with constant time -independent matter density [23, 24]. This assumption is in obvious contradiction to the hydrodynamical equations. In fact, the energy density remains unchanged only if the matter is at rest and the gravitational force, $F \propto \nabla\phi$ vanishes. This inconsistency can in principle be avoided if we consider a static Einstein universe, where the gravitational force of the matter is compensated by the antigravitational force of an appropriately chosen cosmological constant. We consider the fundamental equation of dynamics of a mass point with variable mass. This

is also referred to as the Meshchersky equation.

$$m \frac{d\vec{v}}{dt} = F + \frac{dm}{dt} \vec{u} \quad (1)$$

It should be pointed out that in an inertial frame, \vec{F} is interpreted as the force of interaction of a given body with surrounding bodies. The last term $\frac{dm}{dt} \vec{u}$ is referred to as the reactive force. This force appears as a result of the action that the added or separated mass exerts on a given body. If mass is added, then the $\frac{dm}{dt} > 0$ coincides with the vector \vec{u} . If mass is separated, $\frac{dm}{dt} < 0$, and the vector \vec{R} is oppositely directed to the vector \vec{u} .

We consider a fixed volume element ΔV in Euler (non-co-moving) co-ordinates \vec{x} . After equality, when the cold dark matter starts to dominate in small regions of space, we write:

$$\Delta M_d \frac{d\vec{v}_d}{dt} = -\Delta M_d \cdot \nabla \phi - \nabla p_{b\gamma} \cdot \Delta V - \frac{dM_{b\gamma}}{dt} \vec{u} \quad (2)$$

Here the first term on L. H. S represents the acceleration in the mass of cold dark matter of mass ΔM_d and \vec{v}_d is the velocity of dark matter element. The first term on the R. H. S. represents the gravitational force on the cold dark matter. The second term on right is for the force due to the pressure of the baryon - radiation plasma [31]. The last term on R. H. S. is the reactive force on the cold dark matter due to the ejection of baryon - radiation plasma from regions dominated by cold dark matter. This last force arises only due to the model that we assume here. Here $p_{b\gamma}$ is the pressure of the baryon-radiation plasma and \vec{u} is the ejection velocity of the Baryon - radiation plasma with respect to the concentrated region of cold dark matter. Note that in (eq. 2), we do not take the pressure of cold dark matter into account. This is because, the cold dark matter is highly non - relativistic. Therefore, we neglect its pressure. We assume that after equality, the strongly coupled Baryon - radiation plasma starts to rapidly decouple from the matter. The matter at these epochs is predominantly Cold Dark matter.

After Equality, in regions dominated by highly non - relativistic Cold dark matter, we write the continuity equation for the ejection of baryon- radiation plasma complex :

$$\frac{dM_{b\gamma}}{dt} = \int_{\Delta V} \frac{\partial \varepsilon_{b\gamma}}{\partial t} dV \quad (3)$$

where $M_{b\gamma}$ is the ejecting mass of baryon-radiation plasma and $\varepsilon_{b\gamma}$ is the energy density of baryon radiation plasma in the concentrated region of cold dark matter, from where it is ejecting out. In this model, we assume that the ejection of baryon - radiation plasma out of a concentrated region of heavy Cold dark matter does not disturb the equilibrium of the dark matter. This we can assume only because of the truly collisionless nature of Cold dark matter. This is the fundamental assumption of this model. We neglect the flux of the Cold dark matter out of a region of volume ΔV in the time that the baryon - radiation plasma flows out of this region. The rate of flow is entirely determined by the flux of the baryon - radiation plasma and we write :

$$\frac{dM_{b\gamma}}{dt} = \int_{\Delta V} \frac{\partial \varepsilon_{b\gamma}}{\partial t} dV = - \int_{\Delta V} \vec{\nabla} \cdot \varepsilon_{b\gamma} \vec{u} dV \quad (4)$$

We write (Eq. 2) as:

$$\varepsilon_d \Delta V \frac{d\vec{v}_d}{dt} = -\varepsilon_d \Delta V \nabla \phi - \nabla p_{b\gamma} \Delta V - \frac{\partial \varepsilon_{b\gamma}}{\partial t} \Delta V \vec{u} \quad (5)$$

where ε_d is the energy density of cold dark matter, and ϕ is the gravitational potential of cold dark matter.

$$\varepsilon_d \left[\frac{\partial \vec{v}_d}{\partial t} + \vec{v}_d \cdot \nabla \vec{v}_d \right] = -\varepsilon_d \nabla \phi - \nabla p_{b\gamma} - \frac{\partial \varepsilon_{b\gamma}}{\partial t} \vec{u} \quad (6)$$

Now using the Jeans theory, we introduce small perturbations about the equilibrium values of variables, W. Bonnor[32]:

$$\varepsilon_d(\vec{x}, t) = \varepsilon_{do} + \delta \varepsilon_d(\vec{x}, t) \quad (7)$$

$$\vec{v}_d(\vec{x}, t) = \vec{v}_{do} + \delta \vec{v}_d(\vec{x}, t) = \delta \vec{v}_d(\vec{x}, t) \quad (8)$$

$$\phi(\vec{x}, t) = \phi_o + \delta \phi(\vec{x}, t) \quad (9)$$

$$\vec{u}(\vec{x}, t) = \vec{u}_o + \delta\vec{u}(\vec{x}, t) \quad (10)$$

where $v_{do} \ll c$, the speed of light, $\vec{u}_o \neq 0$ and $\delta\varepsilon_d \ll \varepsilon_{do}$

$$p_{b\gamma}(\vec{x}, t) = p_{b\gamma}(\varepsilon_{b\gamma o} + \delta\varepsilon_{b\gamma}, S_o + \delta S) = p_{b\gamma o} + \delta p_{b\gamma}(\vec{x}, t) \quad (11)$$

$$S(\vec{x}, t) = S_o + \delta S(\vec{x}, t) \quad (12)$$

where S is the entropy of cold dark matter element. Also we write:

$$\delta p_{b\gamma} = c_s^2 \delta\varepsilon_{b\gamma} + \sigma \delta S \quad (13)$$

where c_s^2 is the speed of sound. Neglecting Dissipation, we write:

$$\frac{dS}{dt} = \frac{\partial S}{\partial t} + \vec{v}_d \cdot \vec{\nabla} S \quad (14)$$

$$\nabla^2 \phi = 4\pi G \varepsilon_d \quad (15)$$

From (eq. 4) we write:

$$\frac{\partial \varepsilon_{b\gamma}}{\partial t} + \vec{\nabla} \cdot \varepsilon_{b\gamma} \vec{u} = 0 \quad (16)$$

We substitute the values from (eq. 7) to (eq. 15) in (eq. 16) and (eq. 6) to get:

$$\frac{\partial \delta\varepsilon_{b\gamma}}{\partial t} + \varepsilon_{b\gamma o} \vec{\nabla} \cdot \delta\vec{u} + \vec{\nabla} \cdot (\delta\varepsilon_{b\gamma} \vec{u}_o) = 0 \quad (17)$$

$$\varepsilon_{do} \frac{\partial \delta\vec{v}_d}{\partial t} = -\delta\varepsilon_d \nabla \phi_o - \nabla (c_s^2 \delta\varepsilon_{b\gamma} + \sigma \delta S_{b\gamma}) - \frac{\partial \varepsilon_{b\gamma o}}{\partial t} \delta\vec{u} - \frac{\partial \delta\varepsilon_{b\gamma}}{\partial t} \vec{u}_o \quad (18)$$

$$\nabla^2 \delta\phi = 4\pi G \delta\varepsilon_d \quad (19)$$

$$\frac{d\delta S}{dt} = \frac{\partial \delta S}{\partial t} + (\delta\vec{v}_d \cdot \nabla) S_o = 0 \quad (20)$$

We now take the divergence of (eq. 18) to get:

$$\begin{aligned} \varepsilon_{do} \frac{\partial \vec{\nabla} \cdot \delta\vec{v}_d}{\partial t} &= -(\vec{\nabla} \cdot \delta\varepsilon_d) \nabla \phi_o - \delta\varepsilon_d (4\pi G \varepsilon_{do}) - c_s^2 \nabla^2 \delta\varepsilon_{b\gamma} \\ &\quad - \sigma \nabla^2 \delta S - \vec{\nabla} \cdot \left\{ \frac{\partial (\varepsilon_{b\gamma o} \delta\vec{u})}{\partial t} - \varepsilon_{b\gamma o} \frac{\partial \delta\vec{u}}{\partial t} \right\} - \vec{\nabla} \cdot \left\{ \frac{\partial (\delta\varepsilon_{b\gamma} \vec{u}_o)}{\partial t} - \delta\varepsilon_{b\gamma} \frac{\partial \vec{u}_o}{\partial t} \right\} \end{aligned} \quad (21)$$

Now using (eq. 17) and (eq. 19) we get :

$$\frac{\partial^2 \delta\varepsilon_{b\gamma}}{\partial t^2} - c_s^2 \nabla^2 \delta\varepsilon_{b\gamma} - 4\pi G \varepsilon_{do} \delta\varepsilon_d - (\vec{\nabla} \cdot \delta\varepsilon_d) \nabla \phi_o - \varepsilon_{do} \frac{\partial (\vec{\nabla} \cdot \delta\vec{v}_d)}{\partial t} + \vec{\nabla} \cdot \left\{ \varepsilon_{b\gamma o} \frac{\partial \delta\vec{u}}{\partial t} + \delta\varepsilon_{b\gamma} \frac{\partial \vec{u}_o}{\partial t} \right\} = \sigma \nabla^2 \delta S \quad (22)$$

In a static universe the total energy remains constant. So for small inhomogeneities we write:

$$\delta\varepsilon_{b\gamma} = -\delta\varepsilon_d \quad (23)$$

Thus (eq. 22) becomes:

$$\frac{\partial^2 \delta\varepsilon_{b\gamma}}{\partial t^2} - c_s^2 \nabla^2 \delta\varepsilon_{b\gamma} + 4\pi G \varepsilon_{do} \delta\varepsilon_{b\gamma} + (\vec{\nabla} \cdot \delta\varepsilon_{b\gamma}) \nabla \phi_o - \varepsilon_{do} \frac{\partial \vec{\nabla} \cdot \vec{v}_d}{\partial t} + \vec{\nabla} \cdot \left\{ \varepsilon_{b\gamma o} \frac{\partial \delta\vec{u}}{\partial t} + \delta\varepsilon_{b\gamma} \frac{\partial \vec{u}}{\partial t} \right\} = \sigma \nabla^2 S(x) \quad (24)$$

For strongly -coupled baryon - radiation plasma before recombination, with low baryon densities we write :

$$\varepsilon_{b\gamma} + p_{b\gamma} = \varepsilon_b + \frac{4}{3}\varepsilon_\gamma = \frac{4}{9c_s^2}\varepsilon_\gamma \quad (25)$$

Thus we get:

$$\delta\varepsilon_{b\gamma} + \delta p_{b\gamma} = \frac{4}{9c_s^2}\delta\varepsilon_\gamma \quad (26)$$

The baryon - radiation plasma is only affected by the gravitational potential of the cold dark matter in these epochs. We argue that the baryon-radiation plasma ejecting out of concentrated regions of cold dark matter is always in equilibrium, even in the presence of cold dark matter. This is possible only if the cold dark matter is truly collisionless. So for epochs between equality and recombination, we write:

$$\frac{\partial p_{b\gamma}}{\partial T} = \frac{\varepsilon_{b\gamma} + p_{b\gamma}}{T} \quad (27)$$

The above equation is of profound importance in this model. This is because, it ensures that the equilibrium of the ejecting baryon -radiation plasma is not disturbed, even when it flows out of concentrated regions of cold dark matter. This we can write, because the distribution function of pressure of the baryon-radiation plasma depends only on $\frac{E}{T}$. We assume here that the chemical potential is much smaller than the temperature. So from small perturbations about the equilibrium values of variables $p_{b\gamma}$ and $\varepsilon_{b\gamma}$ in the (eq. 27), about a fixed temperature T, we get,

$$\frac{\delta p_{b\gamma}}{\delta T} = \frac{4\delta\varepsilon_\gamma}{9c_s^2 T} \quad (28)$$

Thus we write:

$$\delta\varepsilon_{b\gamma} = \frac{4\delta\varepsilon_\gamma}{9c_s^2} \left\{ 1 - \frac{\delta T}{T} \right\} \quad (29)$$

Here $\frac{\delta T}{T} \equiv \Theta$, which represents the anisotropy in temperature of radiation for small baryon densities after equality. For small perturbations in cold dark matter, using (eq. 29) in (eq. 24), we write:

$$\begin{aligned} & \frac{4}{9c_s^2} \frac{\partial^2 \delta\varepsilon_\gamma \{1 - \Theta\}}{\partial t^2} - \frac{4}{9} \nabla^2 \{ \delta\varepsilon_\gamma \{1 - \Theta\} \} + \nabla^2 \phi_o \frac{4}{9c_s^2} \delta\varepsilon_\gamma \{1 - \Theta\} + \frac{4}{9c_s^2} \vec{\nabla} \cdot \{ \delta\varepsilon_\gamma (1 - \Theta) \} \nabla \phi_o \\ & - \varepsilon_{do} \frac{\partial(\vec{\nabla} \cdot \delta \vec{v}_d)}{\partial t} + \vec{\nabla} \cdot \{ \varepsilon_{b\gamma o} \frac{\partial \delta \vec{u}}{\partial t} \} + \frac{4}{9c_s^2} \vec{\nabla} \cdot \{ \delta\varepsilon_\gamma (1 - \Theta) \} \frac{\partial \vec{u}_o}{\partial t} = \sigma \nabla^2 \delta S(x) \end{aligned} \quad (30)$$

For small perturbations in Cold dark matter, we neglect the term $\vec{\nabla} \cdot \delta \vec{v}_d$. This is due to cold dark matter being highly non -relativistic. Also we neglect $\vec{\nabla} \cdot \frac{\partial \delta \vec{u}}{\partial t}$. This is due to the fact, that in this model, we assume that the ejecting velocity of baryon - radiation plasma does not suffer interactions and collisions. This is because of dark matter being truly collisionless. So, we further write:

$$\begin{aligned} & \frac{4}{9c_s^2} \frac{\partial^2 \delta\varepsilon_\gamma \{1 - \Theta\}}{\partial t^2} - \frac{4}{9} \nabla^2 \{ \delta\varepsilon_\gamma \{1 - \Theta\} \} + 4\pi G \varepsilon_{do} \frac{4}{9c_s^2} \delta\varepsilon_\gamma \{1 - \Theta\} \\ & + \frac{4}{9c_s^2} \vec{\nabla} \cdot \{ \delta\varepsilon_\gamma (1 - \Theta) \} \nabla \phi_o + \frac{4}{9c_s^2} \vec{\nabla} \cdot \{ \delta\varepsilon_\gamma (1 - \Theta) \} \frac{\partial \vec{u}_o}{\partial t} = \sigma \nabla^2 \delta S(x) \end{aligned} \quad (31)$$

In the above equation, we see the contribution of divergence in the anisotropy of radiation at epochs between equality and recombination. At epochs near recombination, the strongly coupled baryon-radiation plasma starts rapidly to decouple from the cold dark matter. During these epochs, therefore the actual velocity of the baryon -radiation plasma does not change appreciably due to the gravitational force of cold dark matter. During these epochs, the rate of increase of ejection velocity is only due to gravitation potential of cold dark matter. There is no force due to the pressure of the baryon -radiation plasma. Therefore we write:

$$\frac{\partial \vec{u}_o}{\partial t} = -\nabla \phi_o \cdot \quad (32)$$

We can argue that due to high baryon densities, the pressure of the baryon-radiation plasma vanishes at these epochs. But the baryon densities can never be very high. This is because it will hinder hydrogen nucleosynthesis after recombination. Thus it has to be assumed that the pressure of the baryon - radiation plasma vanishes not due to very high baryon densities, but due to a strange form of energy (dark energy) with negative pressure, which has started to dominate near recombination. Therefore, the acceleration in the ejection velocity will be primarily due to the gravitational forces. So, we write:

$$\vec{u}_o = v_{b\gamma o} \vec{v}_o - v_{do} \vec{v}_o \quad (33)$$

$$\dot{\vec{u}}_o = v_{b\gamma o} \dot{\vec{v}}_o - v_{do} \dot{\vec{v}}_o \quad (34)$$

$$v_{b\gamma o} \dot{\vec{v}}_o = 0 \quad (35)$$

The above (eq. 35) shows that the actual velocity of the baryon-radiation plasma stops to increase at epochs, where it is almost about to decouple from cold dark matter. Therefore, at these epochs the acceleration of cold dark matter reverses its sign and we write:

$$v_{do} \dot{\vec{v}}_o = \nabla \phi_o \quad (36)$$

With these Assumptions the (eq. 31) reduces to:

$$\frac{\partial^2}{\partial t^2} \{ \delta \varepsilon_\gamma (1 - \Theta) \} \partial t^2 - c_s^2 \nabla^2 \{ \delta \varepsilon_\gamma (1 - \Theta) \} + 4\pi G \varepsilon_{do} \{ \delta \varepsilon_\gamma (1 - \Theta) \} = \frac{9c_s^2}{4} \sigma \nabla^2 \delta S(x) \quad (37)$$

Considering adiabatic perturbations:

$$\delta S = 0 \quad (38)$$

We use:

$$\delta \varepsilon_\gamma (1 - \Theta)(\vec{x}, t) = \int \delta \varepsilon_{\gamma k}(t) (1 - \Theta)_k(t) e^{i\vec{k} \cdot \vec{x}} \frac{d^3 k}{(2\pi)^{3/2}} \quad (39)$$

Then we write:

$$\delta \varepsilon_{\gamma k}(t) (1 - \theta)_k(t) = y_k(t) \quad (40)$$

The (eq. 37) now reduces to:

$$\ddot{y}_k(t) + k^2 c_s^2 y_k(t) + 4\pi G \varepsilon_{do} y_k(t) = 0 \quad (41)$$

or

$$y_k(t) + (k^2 c_s^2 + 4\pi G \varepsilon_{do}) y_k(t) = 0 \quad (42)$$

The above equation has two solutions: $y_k \propto e^{\pm i\omega(t)}$, where

$$\omega(k) = \sqrt{k^2 c_s^2 + 4\pi G \varepsilon_{do}} = \sqrt{k^2 c_s^2 - 4\pi G \varepsilon_{b\gamma o} \left(1 - \frac{\varepsilon_o}{\varepsilon_{b\gamma o}}\right)} \quad (43)$$

Here the Jeans length is :

$$\lambda_J = \frac{2\pi}{k_J} = c_s \sqrt{\frac{\pi}{G \varepsilon_{b\gamma o} \left\{1 - \frac{\varepsilon_o}{\varepsilon_{b\gamma o}}\right\}}} \quad (44)$$

Thus $\omega(k)$ is real for $\lambda < \lambda_J$ or $k > k_J$, where

$$k_J = \frac{2\pi}{c_s} \sqrt{\frac{G \varepsilon_{b\gamma o} \left\{1 - \frac{\varepsilon_o}{\varepsilon_{b\gamma o}}\right\}}{\pi}} \quad (45)$$

when

$$k^2 c_s^2 > 4\pi G \varepsilon_{b\gamma o} \left\{ 1 - \frac{\varepsilon_o}{\varepsilon_{b\gamma o}} \right\} \quad (46)$$

Where ε_o is the total equilibrium value of energy density and $\varepsilon_{b\gamma o}$ is the equilibrium energy density of baryon-radiation plasma, at these epochs. We interpret from above, that all the modes near recombination are real. This is because the value of $\frac{\varepsilon_o}{\varepsilon_{b\gamma o}}$ starts increasing rapidly near recombination. This is due to the decoupling of baryon - radiation plasma from cold dark matter. This explains the maximum frequency of fluctuations in the CMB temperature anisotropy at red -shifts of recombination. For $\lambda < \lambda_J$ the solutions are:

$$y_k(\vec{x}, t) \propto \sin(\omega t + kx + \alpha) \quad (47)$$

For $\lambda < \tau_\gamma < \lambda_J$, where τ_γ stands for the mean free path of free- streaming photons after equality, free-streaming becomes important. Free-streaming refers to the propagation of photons without scattering. We write the equation for free-streaming photons in the conformal Newtonian gauge [33]. The gravitational potential is primarily due to the cold dark matter. We argue that the photons can still be described by the equilibrium distribution functions. This is because the cold Dark matter is collisionless. Therefore, their mutual interactions will not disturb the equilibrium of the ejecting baryon-radiation plasma. Also, we treat the baryon-radiation plasma as free-streaming photons for low baryon-densities. We use the metric below:

$$ds^2 = -1 - 2\psi(\vec{x}, t)dt^2 + a^2\delta_{ij}(1 + 2\phi(\vec{x}, t))dx^i dx^j \quad (48)$$

where ψ corresponds to the Newtonian potential and ϕ is the perturbation to the spatial curvature. The equation for free- streaming photons is:

$$\frac{1}{p} \frac{dp}{dt} = -H - \frac{\partial\phi}{\partial t} - \frac{\vec{p}^i}{a} \frac{\partial\psi}{\partial x^i} \quad (49)$$

$$\psi = \phi \quad (50)$$

We can make the above assumption, for epochs between equality and recombination. This is because after Equality, and before recombination, the dominant contribution to the potential is due to cold dark matter. So, we neglect the non-diagonal components of the energy-momentum tensor of the baryon-radiation plasma. Recall that the difference $\psi - \phi$ is suppressed compared to ψ , atleast by the ratio of the photon mean free path to the perturbation scale. Thus we assume it to be a single perfect fluid. In a static Einstein universe, we neglect the terms due to the Hubble parameter H . At scales of the order of mean-free path of the free-streaming photons in the baryon-radiation plasma, i. e. of the order of τ_γ , we neglect the spatial inhomogeneities $\frac{\partial\phi}{\partial\vec{x}}$. We therefore write:

$$\frac{1}{p} \frac{dp}{dt} = -\frac{\partial\phi}{\partial t} \quad (51)$$

Using the assumption of (eq. 50), we write:

$$\frac{1}{p} \frac{dp}{dt} = -\frac{\partial\psi}{\partial t} \quad (52)$$

Recall that the Newtonian potential ψ corresponds to the dark matter potential ϕ_{do} . This ϕ_{do} remains constant between epochs of equality and recombination. Thus we write:

$$\log p = -\psi \quad (53)$$

$$\log p = -\phi_{do} \Rightarrow p = e^{-\phi_{do}} \Rightarrow \delta p = -e^{-\phi_{do}} \delta\phi_d \quad (54)$$

The momentum per unit volume of radiation at epochs of decoupling is equivalent to radiation pressure. Also, if the photons, just at the epochs of decoupling are in thermal equilibrium, and that it has same average energy associated with each independent degree of freedom, we write:

$$\delta p_\gamma = \frac{1}{3} \delta p \quad (55)$$

where δp_γ is the increment in radiation pressure associated with each independent degree of freedom at decoupling. Since for a unit volume decoupling photons' energy density is same as radiation pressure per degree of freedom, we write:

$$\delta \varepsilon_\gamma = -\frac{1}{3} e^{-\phi_{do}} \delta \phi_d \quad (56)$$

We use (eq. 40 and (eq. 47) to write:

$$\delta \varepsilon_{\gamma k}(1 - \Theta)(\vec{x}, t) = A \sin(\omega t + kx + \alpha) \quad (57)$$

where

$$\omega = \sqrt{k^2 c_s^2 - 4\pi G \varepsilon_{b\gamma o} \left(1 - \frac{\varepsilon_o}{\varepsilon_{b\gamma o}}\right)} \quad (58)$$

Therefore, for epochs of decoupling, we write:

$$\frac{-e^{-\phi_{do}} \delta \phi_d}{3} \{1 - \Theta\} = A \sin(\omega t + kx + \alpha) \quad (59)$$

or

$$\{\Theta - 1\} = \frac{3e^{\phi_{do}}}{\delta \phi_d} A \sin(\omega t + kx + \alpha) \quad (60)$$

From the above equation we can write:

$$\frac{\delta \phi_d}{3} = \{\theta + \phi_d(1 + \theta)^2\} \quad (61)$$

Therefore, we write:

$$\theta \approx \frac{\delta \phi_d}{3} \quad (62)$$

The above result is the same as that predicted by Sachs-Wolfe. Three types of effects(due to fluctuations in density, velocities and potential)simultaneously contribute to the CMB temperature anisotropy. The fluctuations that matter at scales beyond 1° are those in the gravitational potential $\delta \phi_d$ (Sachs-Wolfe effect)[34].

The above equation represents the anisotropy in the CMB temperature, for epochs near recombination, for regions where sufficient primordial helium synthesis takes place, even before recombination, or the dark energy has started to dominate. Recall that in assumption of(eq. 32), we argue that the force due to pressure of baryon-radiation plasma vanishes near recombination epochs, and that it can be only due to the fact that the dark energy with negative pressure has started to dominate. The value of ϕ_{do} is constant. This is because the potential of cold dark matter remains constant for many epochs between equality and recombination. Thus in a static universe, with this model, we see that the dominant component in CMB temperature anisotropy fluctuations is near recombination. This is because, at epochs near recombination, the perturbations in the cold dark matter potential are very small. This is because in this model, the decoupling of the baryon - radiation plasma from concentrated regions of cold dark matter, near epochs of recombination is almost near completion.

We now write the CMB fluctuations for supercurvature modes i. e modes with $\lambda \gg \lambda_J$, and for regions where sufficient primordial helium synthesis takes place, even before recombination, i. e. the baryon-densities are high. We neglect the effect of gravity at these epochs. We can do so because at late recombination epochs, the pressure of baryon-radiation plasma is low, and the negative pressure of a strange form of energy(dark energy), which starts to dominate at these epochs, cancels the effect of forces due to low pressure baryon-radiation plasma and that of gravity. we therefore write:

$$y_k \propto e^{\pm i k c_s t} \quad (63)$$

Also for $\lambda \gg \lambda_J$, free-streaming of photons is no longer relevant. Therefore the scattering of photons will dilute the anisotropy to a large extent. For late recombination epochs, when the radiation has almost decoupled from matter, we write:

$$c_s^2 = \frac{1}{3} \Rightarrow y_k \propto e^{\pm \frac{i k t}{\sqrt{3}}} \quad (64)$$

We have neglected gravity here again, because at these epochs, the effect of dark energy, which had started to dominate from some earlier epochs is to cancel the forces due to gravity and the pressure of baryon-radiation plasma. This is possible even in regions of lower baryon densities, which has higher pressure than the regions of higher baryon densities. This is only because the dark energy with negative pressure had been dominating from some earlier recombination epochs. The above equation shows that the frequency of fluctuations in CMB temperature anisotropy spectrum of supercurvature modes (modes which enter the horizon very early near recombination epochs), at late recombination, remain constant till today. This is valid both for regions of low or higher baryon densities (where sufficient primordial helium nucleosynthesis takes place before recombination). This is because at very late recombination epochs when the radiation has almost decoupled fully from matter, the speed of sound approaches a constant value of $\frac{1}{3}$. The effect of scattering of photons will dilute this anisotropy in supercurvature modes. So, it is of not much cosmological significance.

III. INSTABILITY IN EXPANDING UNIVERSE: NEWTONIAN THEORY

Using our model (see Sec. I), we study the same scenario, i. e. of cold dark matter in the presence of strongly coupled baryon - radiation plasma at epochs between equality and recombination. Here we use Newtonian theory of expanding universe [32]. We treat the dynamics between cold dark matter and the baryon - radiation plasma, again in the framework of the Meshchersky equation. We assume that the strongly coupled baryon - radiation plasma is in the presence of a gravitational potential. This potential, is only due to cold dark matter, at epochs after equality and before recombination. We neglect the non-diagonal components of the energy-momentum tensor of the cold dark matter. This is because, the difference $\psi - \phi$ is suppressed as compared to ψ , atleast by the ratio of the mean free path to the perturbation scale. We consider dark matter as highly non -relativistic fluid compared to the baryon - radiation plasma. We consider epochs when the Dark matter has already started clustering. We neglect the rate of flow of cold dark matter out of a given region of space, as compared to the baryon-radiation plasma. Therefore we write:

$$\frac{\partial \varepsilon_{b\gamma}}{\partial t} + \vec{\nabla} \cdot (\varepsilon_{b\gamma} \vec{u}) = 0 \quad (65)$$

where \vec{u} is the relative velocity of the baryon - radiation plasma with respect to the cold dark matter. We write the Meshchersky equation for cold dark matter with baryon radiation plasma, ejecting out of concentrated regions of space dominated by cold dark matter:

$$\varepsilon_{do} \left\{ \frac{\partial v_{do}}{\partial t} + (v_{do} \cdot \nabla) v_{do} \right\} = -\varepsilon_{do} \nabla \phi_o - \nabla p_{b\gamma} - \frac{\partial \varepsilon_{b\gamma o}}{\partial t} \vec{u}_o \quad (66)$$

In an expanding flat, Isotropic and homogenous universe, we write:

$$v_{b\gamma} = v_{b\gamma o}(t); \varepsilon_{b\gamma} = \varepsilon_{b\gamma o}(t); \vec{u}_o = H(t) \cdot \vec{x}; \varepsilon_{do} = \varepsilon_{do}(t); v_{do} = v_{do}(t) \quad (67)$$

Therefore we write:

$$\frac{\partial \varepsilon_{b\gamma o}(t)}{\partial t} + \varepsilon_{b\gamma o}(t) \vec{\nabla} \cdot \vec{u}_o = 0 \quad (68)$$

We first discuss for scales $\gg H^{-1}$ i. e the curvature scale. In this case, we neglect the velocities of Cold dark matter particles. This is because, there is insufficient time to move highly non -relativistic cold dark matter upto distances greater than the Hubble scale. Therefore the entropy per cold dark matter is conserved on supercurvature scales ($k\eta > 1$). So we write:

$$\frac{\partial \varepsilon_{b\gamma o}(t)}{\partial t} + 3\varepsilon_{b\gamma o}(t)H(t) = 0 \quad (69)$$

The (eq. 63) gives:

$$\varepsilon_{do} \nabla \phi_o + \nabla p_{b\gamma} + \frac{\partial \varepsilon_{b\gamma o}}{\partial t} \vec{u}_o = 0 \quad (70)$$

We take the divergence of the above equation to write:

$$6H^2 \varepsilon_{b\gamma o} = 4\pi G \varepsilon_{do}^2 \quad (71)$$

To get the above equation, for near recombination epochs, we use:

$$\nabla^2 p_{b\gamma} = 0 \quad (72)$$

We use the above assumption because for epochs near recombination, the baryon-radiation plasma starts to decouple fast. Therefore, only the gravitational force determines the acceleration of dark matter. Now for Adiabatic perturbations, we write:

$$\delta S = 0 \quad (73)$$

Also, we write the following perturbations for other variables:

$$\varepsilon_d = \varepsilon_d = \varepsilon_{do} + \delta\varepsilon_d(\vec{x}, t); \vec{u} = \vec{u}_o + \delta\vec{u}(\vec{x}, t); \phi = \phi_o + \delta\phi; p_{b\gamma} = p_{b\gamma o} + \delta p_{b\gamma} = p_o + c_s^2 \delta\varepsilon_{b\gamma} \quad (74)$$

Where the variables have their usual meanings (see Sec. II). We use (eq. 71) in (eq. 66) and (eq. 68) to write :

$$\frac{\partial \delta\varepsilon_{b\gamma}}{\partial t} = \varepsilon_{b\gamma o} \vec{\nabla} \cdot \delta\vec{u} + \vec{\nabla} \cdot \{\delta\varepsilon_{b\gamma} \vec{u}_o\} = 0 \quad (75)$$

and

$$\varepsilon_{do} \nabla \delta\phi + \delta\varepsilon_d \nabla \phi_o + \nabla \delta p_{b\gamma o} + \frac{\partial \varepsilon_{b\gamma o}}{\partial t} \delta\vec{u} + \frac{\partial \delta\varepsilon_{b\gamma}}{\partial t} \vec{u}_o = 0 \quad (76)$$

We use the Langragian co-ordinates and write:

$$\left\{ \frac{\partial}{\partial t} \right\}_x = \left\{ \frac{\partial}{\partial t} \right\}_q - \vec{u}_o \cdot \nabla_x \quad (77)$$

where

$$\vec{u}_o = H(t)q; x = aq \quad (78)$$

We write:

$$\nabla_x = \frac{1}{a} \nabla_q; \delta = \frac{\delta\varepsilon}{\varepsilon_o} \quad (79)$$

where δ is the fractional amplitude of perturbations. We write (eq. 75) in the Co-moving coordinates, using (eq. 65) and (eq. 77) to get:

$$\delta_{b\gamma} \dot{+} + \frac{\vec{\nabla} \cdot \delta\vec{u}}{a} = 0 \quad (80)$$

also we write the (eq. 76) in the co-moving coordinates to get:

$$\frac{\varepsilon_{do}}{a} \nabla \delta\phi + \frac{\varepsilon_{do}}{a} \delta_d \nabla \phi_o + \frac{c_s^2}{a} \varepsilon_{b\gamma o} \nabla \delta_{b\gamma} + \varepsilon_{b\gamma o} \delta_{b\gamma} \dot{+} + \varepsilon_{b\gamma o} \delta_{b\gamma} + \varepsilon_{b\gamma o} \delta_{b\gamma} - \frac{\vec{u}_o^2}{a} \varepsilon_{b\gamma o} \vec{\nabla} \cdot \delta_{b\gamma} = 0 \quad (81)$$

We write (eq. 69) in co-moving coordinates:

$$\varepsilon_{b\gamma o} \dot{+} = - \frac{3\varepsilon_{b\gamma o}(t)H(t)}{a} \quad (82)$$

from (eq. 71), we get:

$$\varepsilon_{do} \nabla \cdot \delta_d = \frac{6H^2 \varepsilon_{b\gamma o}^2 \nabla \cdot \delta_{b\gamma}}{8\pi G \varepsilon_{do}} \quad (83)$$

We take the divergence of (eq. 76), and use (eqn's. 68-77-79) and (eq. 80) to get:

$$\frac{12H^2 \delta_d}{a} + 3aH \delta_{b\gamma} \dot{+} + \left\{ \frac{c_s^2 - u_o^2}{a} \right\} \nabla^2 \delta_{b\gamma} + \nabla \cdot \delta_{b\gamma} - \nabla \cdot \delta_{b\gamma} \left\{ \frac{H(2u_o + 3)}{a} \right\} = 0 \quad (84)$$

We Neglect the fourth term i. e. $\nabla \cdot \delta_{b\gamma}$. This is because, it is proportional to $\frac{\nabla \cdot \nabla \delta u}{a}$, which is very small. This is due to very small divergence in the perturbations to the ejection velocity of baryon -radiation plasma. With this assumption, again we argue that the cold dark matter is truly collisionless. So, in this model, it does not disturb the equilibrium of the ejecting baryon-radiation plasma. From (eq. 71), we get:

$$8\pi G \varepsilon_{do}^2 \delta_d = 6H^2 \varepsilon_{b\gamma o}^2 \delta_{b\gamma} \quad (85)$$

Using the above, we write (eq. 84) as:

$$\delta_{b\gamma} + \frac{3H^3 \varepsilon_{b\gamma}^2 \delta_{b\gamma}}{8\pi G a^2 \varepsilon_{do}^2} + \left\{ \frac{c_s^2 - u_o^2}{3a^3 H} \right\} \nabla^2 \delta_{b\gamma} + \frac{\nabla \cdot \delta_{b\gamma} H^2}{3a^2} \{2u_o + 3\} = 0 \quad (86)$$

Therefore, neglecting the term containing $\nabla \cdot \delta_{b\gamma}$, we write:

$$\delta_{b\gamma} + \frac{3H^3 \varepsilon_{b\gamma}^2 \delta_{b\gamma}}{8\pi G a^2 \varepsilon_{do}^2} + \left\{ \frac{c_s^2 - u_o^2}{3a^3 H} \right\} \nabla^2 \delta_{b\gamma} = 0 \quad (87)$$

We use:

$$\delta_{b\gamma} = \int \delta_{b\gamma k}(t) e^{i\vec{k} \cdot \vec{q}} \frac{d^3 k}{\sqrt{(2\pi)^3}} \quad (88)$$

Thus, we write the equation for the evolution of fractional amplitudes of perturbations in the baryon -radiation plasma as:

$$\delta_{b\gamma} = \exp\left\{k^2 \left(\frac{c_s^2 - u_o^2}{3a^3 H}\right) - \frac{3H^3 \varepsilon_{b\gamma o}^2}{8\pi G a^2 \varepsilon_{do}^2}\right\} t \quad (89)$$

For very late recombination epochs, the radiation starts to decouple rapidly from matter. Therefore, baryon densities are very low. So we write:

$$c_s^2 = \frac{1}{3} \quad (90)$$

where c_s^2 is the speed of sound.

The (eq. 89) shows, that for low baryon densities, fractional amplitudes of perturbations in the radiation, which originate at late recombination epochs grow at the fastest rate. This is because the value of c_s^2 is maximum at late recombination epochs. It is because of the lowest baryon densities, in the coupled baryon -radiation plasma at these epochs. Also we can see that the second term in the exponent in (eq. 89) vanishes for very late recombination epochs, as ratio $\frac{\varepsilon_{b\gamma o}^2}{\varepsilon_{do}^2} \rightarrow 0$. We thus write the equation for evolution of the fractional amplitude of perturbations in the Cold dark matter, which originate after equality, when the clustering of dark matter had already started.

$$\delta_d = \frac{3H^2 \varepsilon_{b\gamma o}^2}{4\pi G \varepsilon_{do}^2} \exp\left\{k^2 \left(\frac{c_s^2 - u_o^2}{3a^3 H}\right) - \frac{3H^3 \varepsilon_{b\gamma o}^2}{8\pi G a^2 \varepsilon_{do}^2}\right\} t \quad (91)$$

The above equation represents the growth of the fractional amplitudes of perturbations in the cold dark matter, which originate after equality, for scales $\gg H^{-1}$. If baryons contribute a significant factor of the total matter density, CDM growth rate will be slowed down between equality and the recombination epochs [35]. Also we see that the perturbation growth rate will slow with scale factor [36]. The CDM density fluctuations will dominate the density perturbations of baryon-radiation plasma [35]. This is because for scales $\gg H^{-1}$, the density perturbations of baryon-radiation plasma are washed out by the scattering of photons at scales $> \tau_\gamma$, which is the mean free path of photons. The perturbations in the cold dark matter will cease to grow when the dark energy starts to dominate [37]. We can interpret this from (eq. 91). This is because with the growth of dark energy, the value of c_s^2 will decrease. This will then lead to ceasing of growth of CDM perturbations. There is existence of non-linear structures today. This implies that the growth of fluctuations must have been driven by non-baryonic dark matter, which was not relativistic at recombination. Also, we see that the perturbations at supercurvature scales grow slowly. Recall that these are the modes which enter the horizon very early, well before the recombination epochs. The slow growth of such modes is because, it is only at late recombination epochs that the second term in the exponent in (eq. 91) will vanish due to $\frac{\varepsilon_{b\gamma o}^2}{\varepsilon_{do}^2} \rightarrow 0$. Also, it is only at late recombination epochs, that the value of c_s^2 reaches its maximum value of $\frac{1}{3}$, just

before decoupling. The amplitude of the fractional density perturbations in the cold dark matter, in (eq. 91) will be maximum when the ratio $\frac{\varepsilon_{b\gamma o}}{\varepsilon_{do}} \rightarrow 1$. This will occur when

$$\varepsilon_{do} = \frac{3H^2}{2\pi G} \quad (92)$$

In writing the above equation, we use the result of (eq. 71). So for epochs when the density of baryon-radiation plasma is equal to the density of the cold dark matter, (eq. 91) is:

$$\delta_d = \frac{\varepsilon_{do}}{2} \exp\{k^2(\frac{c_s^2 - u_o^2}{3a^3H}) - \frac{H\varepsilon_{do}}{4a^2}\}t \quad (93)$$

Now we discuss the gravitational instability for scales $< H^{-1}$. This originates after equality with dark matter dominance in the presence of strongly coupled baryon - radiation plasma in an expanding universe. We discuss it in the Newtonian theory. The decoupling of the strongly coupled baryon - radiation plasma from the non-relativistic cold dark matter starts after equality. Let us assume that the separation of this plasma from the cold dark matter gives a relative velocity of \vec{u}_o to the baryon - radiation plasma. At small scales with low baryon densities, we neglect the contribution of non-diagonal components in the energy- momentum tensor of dark matter. Therefore, we treat it as a perfect fluid for many epochs between equality and recombination. We treat the strongly coupled baryon - radiation plasma as a perfect fluid for epochs between equality and recombination. This is because of low baryon -densities at these epochs. Recall that the baryon densities only starts increasing substantially after recombination. This is when the primordial nucleosynthesis of hydrogen and helium will start. However, as an exception in certain regions, the primordial nucleosynthesis may start at epochs before recombination. We now conclude that there is sufficient time for the Cold dark matter to flow through distances at scales $< H^{-1}$. This is because from (eq. 91), we see that the perturbations in the cold dark matter, at small scales, grows at the fastest rate. Therefore we write the Meshchersky equation as below:

$$\varepsilon_{do}\{\frac{\partial \vec{v}_{do}}{\partial t} + (\vec{v}_{do} \cdot \nabla)\vec{v}_{do}\} = -\varepsilon_{do}\nabla\phi_o - \nabla p_{b\gamma} - \frac{\partial \varepsilon_{b\gamma o}}{\partial t}\vec{u}_o \quad (94)$$

where

$$\vec{u}_o = \vec{v}_{b\gamma o} - \vec{v}_{do} \quad (95)$$

At small scales, we assume that in the time that the baryon -radiation plasma flows out of a given region of space, the inhomogeneity in Cold dark matter in that time duration is negligible. So we assume that the baryon-radiation plasma can flow out of a concentrated region of dark matter without generating the collision terms. This is because the cold dark matter is highly non-relativistic and collisionless. Also, because the dark matter has already started to cluster. Therefore we write:

$$\vec{v}_{b\gamma} = \vec{v}_{b\gamma o}(t) = H(t)\vec{x} \quad (96)$$

where \vec{x} is the eulerian coordinate. We write the continuity equation for flow of the strongly coupled baryon - radiation plasma as :

$$\frac{\partial \varepsilon_{b\gamma}}{\partial t} + \nabla \cdot (\varepsilon_{b\gamma} \vec{v}_{b\gamma}) = 0 \quad (97)$$

Here $\varepsilon_{b\gamma}$ is equal to $\varepsilon_{b\gamma o}(t)$

$$\dot{\varepsilon}_{b\gamma o} = -3H\varepsilon_{b\gamma o} \quad (98)$$

We take the divergence of (eq. 94) and neglect the spatial dependence of \vec{u}_o . This is due to the collisionless nature of dark matter. We also assume that ε_{do} is constant for small scales. We thus write the Friedmann equation:

$$\dot{H} + H^2 = -\frac{4\pi G\varepsilon_{do}}{3} \quad (99)$$

For adiabatic perturbations we write from :

$$\begin{aligned} \varepsilon_d(\vec{x}, t) &= \varepsilon_{do} + \delta\varepsilon_d(\vec{x}, t); \vec{v}_d(\vec{x}, t) = \vec{v}_{do} + \delta\vec{v}_d(\vec{x}, t) = \delta\vec{v}_d(\vec{x}, t); \phi(\vec{x}, t) = \phi_o + \delta\phi(\vec{x}, t); \\ \vec{u}(\vec{x}, t) &= \vec{u}_o + \delta\vec{u}(\vec{x}, t); p_{b\gamma} = p_{b\gamma o} + c_s^2\delta\varepsilon_{b\gamma}; \delta S = 0 \end{aligned} \quad (100)$$

where the variables have their usual meanings (see Sec. II). Then we use the perturbed values of variables in (eq. 94), to write:

$$\begin{aligned} \varepsilon_{do}\delta v_{b\gamma} - \varepsilon_{do}\dot{\delta\vec{u}} + \delta\varepsilon_d v_{b\gamma o} - \delta\varepsilon_d \dot{\vec{u}} + \varepsilon_{do}(v_{b\gamma o} \cdot \nabla)\delta v_{b\gamma} - \varepsilon_{do}\vec{u}_o \cdot \nabla\delta v_{b\gamma} - \varepsilon_{do}v_{b\gamma o} \cdot \nabla\delta\vec{u} \\ + \varepsilon_{do}\vec{u}_o \cdot \nabla\delta\vec{u} = -\varepsilon_{do}\nabla\delta\phi - \delta\varepsilon_d\nabla\phi_o - c_s^2\nabla\varepsilon_{b\gamma} - \varepsilon_{b\gamma o}\dot{\delta\vec{u}} - \delta\varepsilon_{b\gamma}\vec{u}_o \end{aligned} \quad (101)$$

We then use the perturbed values of variables in (eq. 97) to get:

$$\delta\varepsilon_{b\gamma} + \varepsilon_{b\gamma o}(t)\nabla \cdot \delta v_{b\gamma} + v_{b\gamma o}\nabla \cdot \delta\varepsilon_{b\gamma} = 0 \quad (102)$$

We now write (eq. 98) in co-moving coordinates.

$$\varepsilon_{b\gamma o} = -\frac{3\varepsilon_{b\gamma o}H}{a} \quad (103)$$

We also write (eq. 102) in co-moving coordinates to get:

$$\delta\varepsilon_{b\gamma} = -\frac{\varepsilon_{b\gamma o}}{a}\nabla \cdot \delta v_{b\gamma} \quad (104)$$

Using (eq. 95) in (eq. 94) and also using the (eq. 103), (eq. 104), we write (eq. 94) in co-moving coordinates. We then take the divergence of the obtained equation and use the results of (eq. 112) and (eq. 117), to write:

$$\begin{aligned} \delta_{b\gamma}'' + H\left\{1 + \frac{3}{a} - \frac{u_o}{aH}\right\}\delta_{b\gamma}' - \left\{\frac{3\dot{H}}{a} + \frac{3H^2\varepsilon_{b\gamma o}}{\varepsilon_{do}} + \frac{9H^2\varepsilon_{b\gamma o}}{a\varepsilon_{do}} - \frac{3Hu_o}{a^2} + 4\pi G\varepsilon_{b\gamma o} - \frac{4\pi G\varepsilon_{b\gamma o}}{a}\right\}\delta_{b\gamma} \\ - \frac{3H\varepsilon_{b\gamma o}}{a^2\varepsilon_{do}}\{av_{b\gamma o} - 1 - u_o\}\nabla \cdot \delta v_{b\gamma} - \frac{\varepsilon_{b\gamma o}\nabla \cdot \delta_{b\gamma}'}{a\varepsilon_{do}} + \frac{\varepsilon_{b\gamma o}}{\varepsilon_{do}}\left\{\frac{c_s^2 - v_{b\gamma o}u_o}{a^2}\right\}\nabla^2\delta_{b\gamma} = 0 \end{aligned} \quad (105)$$

Also we can write:

$$\delta\varepsilon_{b\gamma} = \frac{4c_s^2\delta\varepsilon_\gamma(1-\theta)}{9} \quad (106)$$

where $\theta = \frac{\delta T}{T}$, represents the anisotropy in the temperature of radiation. The total energy distribution composed of the sum of the dark matter and the baryon-radiation plasma is a function of time in an expanding universe. If this remains smooth inspite of the inhomogeneities at scales smaller than H^{-1} , we can write:

$$\varepsilon_{b\gamma}a^4(t) + \varepsilon_d a^3(t) = E_{total}(t) \quad (107)$$

or for any time t' , we can write:

$$-a\delta\varepsilon_{b\gamma} = \delta\varepsilon_d \quad (108)$$

At epochs near recombination, when the baryon-radiation plasma is decoupled from the cold dark matter, we assume that their bulk velocity does not appreciably change in these epochs. Therefore, it remains constant for epochs near recombination. The ejection velocity of baryon-radiation plasma is affected only due to the gravitational force of the dark matter potential. There is no force due to the pressure of the baryon-radiation plasma. This can be either due to low pressure of the plasma, due to higher baryonic densities, or due to some new force of strange energy(dark energy), having negative pressure. The baryon densities can never be very high before recombination. This is because it would hinder future hydrogen nucleosynthesis after recombination. So, it would be rightly concluded that a new form of dark energy having negative pressure starts to dominate at these epochs. So, we write:

$$v_{b\gamma o} = \text{constant} \implies \dot{v}_{b\gamma} = 0 \quad (109)$$

$$\dot{v}_{do} = \nabla\phi_o \quad (110)$$

The above equation comes from the same argument, which we give in (eq. 36). The actual velocity of the baryon-radiation plasma stops to increase, when it is almost about to decouple from dark matter. At these epochs, the acceleration of dark matter will reverse its sign. Using the above equation and (eq. 95), we write:

$$\dot{\vec{u}}_o = -\nabla\phi_o \quad (111)$$

and in co-moving coordinates,

$$\dot{\vec{u}}_o = -\frac{\nabla\phi_o}{a} \quad (112)$$

$$\nabla.\delta\dot{u} = -\nabla\delta\phi \quad (113)$$

and in co-moving coordinates,

$$\nabla.\delta\dot{u} = -\frac{\nabla\delta\phi}{a} \quad (114)$$

or

$$\nabla.\delta\dot{u} = -\frac{\nabla^2\delta\phi}{a} = -\frac{4\pi G\delta\varepsilon_d}{a} \quad (115)$$

Using (eq. 103), we can write the above equation as:

$$\nabla.\delta\dot{u} = 4\pi G\varepsilon_{b\gamma o}\delta_{b\gamma} \quad (116)$$

Also we assume,

$$\nabla.\dot{\delta u} = 0 \quad (117)$$

For scales much smaller than the Jeans length $\lambda < \tau_\gamma < \lambda_J$ i. e. scales much smaller than the curvature scale i. e. for which H^{-1} is dominant, we neglect the terms $\propto H$ and write Eq. 105 as:

$$\ddot{y}_k - \frac{u_o}{a}\dot{y}_k + \left\{4\pi G\varepsilon_{b\gamma o}\left(1 - \frac{1}{a}\right) - k^2\varepsilon_{b\gamma o}\left(\frac{c_s^2 - v_{b\gamma o}u_o}{a^2}\right) - \frac{\dot{u}_o}{a}\right\}y_k = 0 \quad (118)$$

We use (eq. 39) and ((eq. 40) in writing the above equation. If we now choose:

$$\begin{aligned} 2\lambda &= -\frac{u_o}{a} \\ \left\{4\pi G\varepsilon_{b\gamma o}\left(1 - \frac{1}{a}\right) - k^2\varepsilon_{b\gamma o}\left(\frac{c_s^2 - v_{b\gamma o}u_o}{a^2}\right) - \frac{\dot{u}_o}{a}\right\} &= \omega^2 \end{aligned} \quad (119)$$

Then the auxillary equation is:

$$D^2 + 2\lambda D + \omega^2 = 0 \quad (120)$$

with

$$D = -\lambda \pm \sqrt{\lambda^2 - \omega^2} \quad (121)$$

The only solution which is of physical significance is when $\lambda < \omega$, i. e. when the roots of the auxillary equation are imaginary, i. e. :

$$\begin{aligned} D &= -\lambda + i\alpha \\ \alpha^2 &= \omega^2 - \lambda^2 \\ y_k &= A\sqrt{\left\{1 + \left(\frac{\lambda}{\alpha}\right)^2\right\}}e^{-\lambda t}\cos\left\{\alpha - \arctan\frac{\lambda}{\alpha}\right\} \end{aligned} \quad (122)$$

Thus we write:

$$\delta\varepsilon_\gamma\{1 - \Theta\} = A\sqrt{\left\{1 + \left(\frac{\lambda}{\alpha}\right)^2\right\}}e^{-\lambda t}\cos\left\{\alpha - \arctan\frac{\lambda}{\alpha}\right\} \quad (123)$$

for $\lambda < \tau_\gamma < \lambda_J$, we can write for free -streaming photons:

$$\frac{1}{p}\frac{dp}{dt} = -H - \frac{\partial\phi}{\partial t} - \frac{p^i}{a}\frac{\partial\psi}{\partial x^i} \quad (124)$$

where ψ corresponds to the Newtonian potential in Perturbed FRW universe. Here we assume,

$$\phi = \psi \quad (125)$$

This is because the potential is dominated by the cold dark matter between epochs of equality and recombination. We neglect the spatial inhomogeneities in the Newtonian potential at very small scales and terms $\propto H$. Using the same argument that we give for (eq. 56.), we get:

$$\delta\varepsilon_\gamma = \frac{1}{3}\delta p = -\frac{e^{-\phi_{do}}\delta\phi_d}{3} \quad (126)$$

Using (eq. 123) and (eq. 126), we write:

$$\frac{-e^{-\phi_{do}}\delta\phi_d\{1-\Theta\}}{3} = A\sqrt{\{1+(\frac{\lambda}{\alpha})^2\}}e^{-\lambda t}\cos\{\alpha - \arctan\frac{\lambda}{\alpha}\} \quad (127)$$

or

$$\{\Theta - 1\} = \frac{3Ae^{\phi_{do}}}{\delta\phi_d}\sqrt{\{1+(\frac{\lambda}{\alpha})^2\}}e^{-\lambda t}\cos\{\alpha - \arctan\frac{\lambda}{\alpha}\} \quad (128)$$

This equation is of same form as the one we get for Einstein's static universe in Jeans theory. In the static universe, we arrived at this equation by an assumption of (eq. 32). This we reasoned, may be partly possible where baryon densities are higher due to early primordial synthesis of Helium before recombination. But the primary reason would be that dark energy, with negative pressure starts to dominate from these epochs. The similarity in form of the two equations, shows that the epochs, for which the CMB temperature anisotropy derived from the solutions in the static Einstein universe and those from the expanding Newtonian match, will be the epochs when the dark energy starts to dominate. We see that the amplitude in (eq. 123) is variable, but at epochs when the temperature anisotropy of radiation is dominant, which is the late recombination epochs, the term λ can be neglected. So we write (eq. 123) as:

$$\{\Theta - 1\} = \frac{3Ae^{\phi_{do}}}{\delta\phi_d}\cos\alpha \quad (129)$$

Using the result of (eq. 61), we write:

$$\theta \approx \frac{\delta\phi_d}{3} \quad (130)$$

The equation for fractional amplitudes of perturbations in the cold dark matter that originate at scales smaller than the mean free path of the photons is:

$$\delta_d = -\frac{4Ac_s^2a}{9\varepsilon_{do}}\sqrt{\{1+(\frac{\lambda}{\alpha})^2\}}e^{-\lambda t}\cos\{\alpha - \arctan\frac{\lambda}{\alpha}\} \quad (131)$$

and for late recombination epochs, neglecting λ , we can write:

$$\delta_d = -\frac{4Ac_s^2a}{9\varepsilon_{do}}\cos\alpha \quad (132)$$

where

$$\alpha = \sqrt{\{4\pi G\varepsilon_{b\gamma o}(1 - \frac{1}{a}) - k^2\varepsilon_{b\gamma o}(\frac{c_s^2 - v_{b\gamma o}u_o}{a^2}) - \frac{\dot{u}_o}{a}\} - (\frac{-u_o}{2a})^2} \quad (133)$$

The above equation shows that the fractional amplitudes of perturbations in cold dark matter at small scales after equality keep growing with scale factor, but with a decreasing frequency of oscillations.

IV. CONCLUSION

We conclude that this model, wherein we assume that the baryon -radiation plasma has an ejection velocity, flowing out of regions of highly non- relativistic cold dark matter, explains the effect of cold dark matter perturbations on CMB temperature anisotropy fluctuations fairly well. The results of (eq. 62) and (eq. 130) represent the Sachs-Wolfe effect. It also correctly predicts the growth of fractional amplitudes of perturbations in cold dark matter, which originate between equality and recombination epochs. The (eq. 59) and (eq. 127) describe the affect of the perturbations in the cold dark matter on the CMB temperature anisotropy fluctuations. They also highlight that the dominant component of CMB temperature anisotropy fluctuations is at red-shifts of very late recombination epochs $z \approx 1100$. This is because the perturbations in the cold dark matter are minimal at these epochs. The unperturbed potential ϕ_{do} remains frozen at epochs between equality and recombination. Therefore the fluctuations in the CMB temperature anisotropy between epochs of equality and recombination are determined by the perturbations in the potential of cold dark matter potential only. The assumptions of (eq. 32) and (eq. 111), assume the growth of a strange form of energy(dark energy), with negative pressure, which cancels the force due to the pressure of baryon-radiation plasma.

These assumptions also represent the fact that the rate of increase of ejection velocity of the Baryon -radiation plasma is only due to the gravitational field of the Cold dark matter. It can be inferred that the force due to its pressure can vanish only to some extent due to low pressure in high baryonic density regions. This is due to early primordial helium nucleosynthesis in such regions. Since the baryonic densities can never be very high before recombination. This is because it will hinder the hydrogen nucleosynthesis which starts after recombination. Therefore, it has to be assumed that the primary reason for the vanishing of pressure of baryon-radiation plasma, at near recombination epochs, is that the dark energy term with negative pressure starts to dominate at these epochs. The epochs when the solutions of the (eq. 59) and (eq. 127) match will give the correct time scales when the Dark energy starts to dominate. This work can be taken up in a following paper. The use of (eq. 27) for strongly coupled baryon -radiation plasma indicates its equilibrium at all epochs between equality and recombination, even in the presence of dark matter. This assumption can only be made if the dark matter is truly collisionless. The correct explanation of predominant contribution to CMB temperature anisotropy from very late recombination epochs, and the Sachs-Wolfe effect, based on the assumptions of (eq. 27), emphasizes the truly collisionless nature of cold dark matter.

Thus it can be concluded that the use of this model for describing the dynamics of Cold dark matter, in presence of strongly coupled Baryon -radiation plasma for epochs between equality and recombination is sufficiently useful. More accurate results can be obtained by studying the dynamics of baryon -radiation plasma using energy momentum tensor of an imperfect fluid in such a scenario. This will be attempted in a future work.

Acknowledgments

We wish to thank Md. Mahfoozul Haque for valuable advice in formatting of the manuscript in Latex. Also, we thank Zaheer and Divyendu for valuable discussions and reading the manuscript.

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